



丘成桐数学科学中心  
YAU MATHEMATICAL SCIENCES CENTER



# 高阶Painlevé方程与Garnier系统

Higher Painlevé equations and Garnier systems

**April 7-11, 2025**

**Room A-103, TSIMF**

## 【 组织者 ORGANIZERS 】

Anton Dzhamay, Beijing Institute of Mathematical Sciences and  
Applications(BIMSA)

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# Contents

|                                 |    |
|---------------------------------|----|
| About the conference.....       | 1  |
| Schedule .....                  | 4  |
| April 7, 2025 - Monday.....     | 5  |
| April 8, 2025 - Tuesday.....    | 6  |
| April 9, 2025 - Wednesday ..... | 7  |
| April 10, 2025 - Thursday.....  | 8  |
| April 11, 2025 - Friday.....    | 9  |
| Titles and Abstracts.....       | 10 |
| Frank Nijhoff.....              | 11 |
| Robert Conte .....              | 11 |
| Xin Wang.....                   | 11 |
| Tomoyuki Takenawa.....          | 12 |
| Teruhisa Tsuda.....             | 12 |
| Alexander Stokes.....           | 12 |
| Anton Dzhamay .....             | 13 |
| Andrei Marshakov .....          | 13 |
| Oleg Chalykh .....              | 13 |
| Di Yang.....                    | 13 |
| Rod Halburd .....               | 14 |
| Hajime Nagoya.....              | 14 |
| Guofu Yu .....                  | 14 |
| Ying-ying Sun.....              | 14 |
| Xiangke Chang.....              | 15 |
| Yik Man Chiang.....             | 15 |
| Zhaowen Yan.....                | 15 |
| Qian Tang.....                  | 15 |
| Yuancheng Xie.....              | 16 |
| Xiaomeng Xu.....                | 16 |
| Ruguang Zhou .....              | 16 |
| Cheng Zhang.....                | 17 |

|   |           |
|---|-----------|
| Lun Zhang.....                                    | 17        |
| Maksim Pavlov .....                               | 17        |
| Zhiwei Wu .....                                   | 17        |
| <b>Welcome to TSIMF.....</b>                      | <b>18</b> |
| <b>About Facilities.....</b>                      | <b>20</b> |
| Registration.....                                 | 20        |
| Guest Room .....                                  | 20        |
| Library .....                                     | 21        |
| Restaurant.....                                   | 21        |
| Laundry .....                                     | 22        |
| Gym .....   | 22        |
| Swimming Pool.....                                | 22        |
| Free Shuttle Bus Service at TSIMF.....            | 22        |
| Playground.....                                   | 22        |
| Contact Information of Administration Staff ..... | 23        |

The background is an abstract, fluid composition of colors. It features deep blues and purples that blend and flow together, creating a sense of movement and depth. The colors are most vibrant in the lower right and upper left corners, fading into darker tones towards the center.

***About the conference***

## 高阶 Painlevé 方程与 Garnier 系统

### Higher Painlevé equations and Garnier systems

#### Date

April 7-11, 2025

#### Venue

Room A-103, TSIMF

#### Organizers

Anton Dzhamay, Beijing Institute of Mathematical Sciences and Applications(BIMSA)

Frank W. Nijhoff, University of Leeds

Da-jun Zhang( 张大军 ), Shanghai University

#### Abstract

Discrete and continuous Painlevé equations have attracted a lot of attention in recent decades, since they define (new) transcendental functions, and exhibit rich mathematical structures, in algebraic geometry, representation theory and asymptotic analysis. The higher analogues of the Painlevé equations, including the isomonodromic Garnier systems have an even richer structure, involving connections with multivariate special functions, including higher-genus Abelian functions, and multiple orthogonal polynomials and an expected higher-dimensional variant of the underlying algebraic geometry that was established for the usual Painlevé equations. The connections with integrable systems, via higher-order similarity reduction, may form a key to a further understanding of these more complicated systems, but so far the study of Garnier and higher Painlevé systems has been lagging behind. One of the aims of the workshop is to repair this imbalance. By bringing together experts as well as interested researchers, we aim at creating a platform where many of the open questions can be discussed and begin to be tackled. Thus, we hope the workshop can act as a launching pad for opening a systematic research program into these systems.

#### Description of the aim

##### Background:

In recent decades the Painlevé equations, and their discrete analogues have been studied extensively, both from the point of view of integrable systems as well as in physics (random matrix models and statistical mechanics) and in algebraic geometry (rational surfaces of initial conditions) and representation theory (affine Weyl groups). In contrast the higher order Painlevé equations have not attracted (yet) a similar level of attention. These higher order ordinary differential and difference equations emerged as multi-phase similarity reductions from integrable hierarchies (in the continuous case), as well as from constructions from integrable partial difference equations (in the discrete case). They are of interest, as they are expected to yield novel transcendental functions which asymptotically go to higher-genus Abelian functions, whereas the usual Painlevé equations tend to elliptic functions (genus one) in the long-time range.

Garnier in 1912 constructed a higher analogue of Fuchs' isomonodromic deformation problem for Painlevé VI with multiple moving singularities and multiple dependent variables. This results in

coupled systems of 2nd order ODE's, compatible through a system of linear PDEs, which can be viewed as the PVI hierarchy. Apart from the Painlevé the property of this higher order system, the limiting behaviour of the solutions lead to hyper-elliptic integrals. The Garnier system remained relatively unexplored until the work by Okamoto in the 1970s, who focused on its Hamiltonian aspects. It has special solutions in terms of multivariate hypergeometric functions (Lauricella, etc.) (see e.g. the monograph "From Gauss to Painlevé" by Iwasaki et al.). More recently a q-Garnier system (Sakai, 2005), and an elliptic variant (Ormerod & Rains, 2015) were established, while reductions from KdV and Boussinesq systems to discrete and continuous Garnier systems were given (Nijhoff & Walker, 2001; Tongas & Nijhoff, 2005), and the isomonodromy problem was studied (Dubrovin & Mazzocco, 2000). Some classification results on algebraic solutions of Garnier systems were obtained (Diarra & Loray, 2015) and in recent years some quantum Painlevé and Garnier systems have been investigated (Nagoya et al, 2004 and 2008, Novikov & Sukeimanov, 2016). Apart from these isolated results, the study of the Garnier systems and higher rank Schlesinger systems (isomonodromic matrix systems) has remained relatively sparse.

### Objectives:

The workshop aims at stimulating the research into the higher Painlevé equations and Garnier systems, by bringing together experts as well as interested researchers and bring to the fore open problems, challenges and possible new directions. This is meant to be a mostly explorative venture in the hope that some synergies can bring about progress in this largely not yet developed area. The following directions will be highlighted:

- Reduction from integrable PDEs and PΔEs: While continuous and discrete Painlevé equations often arise as reductions from integrable partial difference and differential equations, for Garnier systems this remains mostly to be established. Isolated precedents comprise Garnier systems derived from higher-order similarity reduction of continuous & discrete KdV and Boussinesq hierarchies.
- Lagrangian multiform aspects: While the second order discrete and continuous Painlevé equations possess a conventional Lagrangian description, the newly established Lagrangian multiform theory (Lobb & Nijhoff, 2009), which provides a natural variational formalism for multi-time integrable systems, is directly applicable to the case of Garnier systems.
- Connection with higher-genus abelian functions: 'Garnier transcendents' tend to higher-genus Abelian functions in asymptotic limits. The study of the singularity structure of those solutions may help to link the isomonodromy theory to algebra-geometric techniques on Riemann surfaces.
- Special solutions in terms of multivariate hypergeometric functions: For special parameter values of the Garnier systems solutions exist in terms of multivariate hypergeometric functions. This remains to be done for discrete Garnier systems leading possibly to multivariate elliptic hypergeometric functions;
- Algebraic geometry of spaces of initial conditions; Lifting the celebrated work by Sakai on the classification of discrete and continuous Painlevé equations within the context of the algebraic geometry of rational surfaces and affine Weyl groups, to the case of Garnier systems. Some work has begun (e.g. Takenawa, 2024), but requires further development.
- Applications in random matrix theory and physics: Continuous and most distinctly discrete Painlevé equations have played an important role in random matrix ensembles and in the theory of semi-classical orthogonal polynomials. These relations were often motivated from physics, e.g. in 2D quantum gravity and string theory. So far, there have been little appearance of Garnier systems in this context, but the structures are ready to be explored for such connections.

# Schedule

| <i>Time&amp;Date</i> | <i>Monday<br/>(April 07)</i> | <i>Tuesday<br/>(April 08)</i> | <i>Wednesday<br/>(April 09)</i> | <i>Thursday<br/>(April 10)</i> | <i>Friday<br/>(April 11)</i> |
|----------------------|------------------------------|-------------------------------|---------------------------------|--------------------------------|------------------------------|
| 7:30~8:30            | Breakfast                    |                               |                                 |                                |                              |
| Chair                | A Dzhamay                    | DJ Zhang                      | YY Sun                          | C Zhang                        | YC Xie                       |
| 9:00-9:50            | F Nijhoff                    | A Marshakov                   | XK Chang                        | Q Tang                         | M Pavlov                     |
|                      |                              | Group Photo                   |                                 |                                |                              |
| 9:50-10:20           | Coffee Break                 |                               |                                 |                                |                              |
| Chair                | L Zhang                      | RG Zhou                       | Q Tang                          | ZW Yan                         | F Nijhoff                    |
| 10:20-11:05          | R Conte                      | O Chalykh                     | YM Chiang                       | YC Xie                         | ZW Wu                        |
| 11:10-11:55          | X Wang                       | D Yang                        | ZW Yan                          | XM Xu                          |                              |
| 12:00-13:30          | Lunch (90 minutes)           |                               |                                 |                                |                              |
| Chair                | YM Chiang                    | X Wang                        | Free Discussion<br>13:30-17:00  | ZW Wu                          |                              |
| 14:30-15:15          | T Takenawa                   | R Halburd                     |                                 | RG Zhou                        |                              |
| 15:15-16:00          | T Tsuda                      | H Nagoya                      |                                 | C Zhang                        |                              |
| 16:00-16:30          | Coffee Break                 |                               |                                 | Coffee Break                   |                              |
| Chair                | D Yang                       | XK Chang                      |                                 | XM Xu                          |                              |
| 16:30-17:15          | A Stokes                     | GF Yu                         |                                 | L Zhang                        |                              |
| 17:15-18:00          | A Dzhamay                    | YY Sun                        |                                 |                                |                              |
| 18:00-19:30          | Dinner                       |                               | Banquet<br>18:00-20:00          | Dinner                         |                              |

**April 7, 2025 - Monday**

| Time        | Name              | Title  |
|-------------|-------------------|--|
| 7:30~8:30   | Breakfast         |  |
| Chair       | Anton Dzhamay     |  |
| 9:00-9:50   | Frank Nijhoff     | On isomonodromic deformation problems on the torus associated with Lax pairs for elliptic integrable lattice systems |
| 9:50-10:20  | Coffee Break      |  |
| Chair       | Lun Zhang         |  |
| 10:20-11:05 | Robert Conte      | A holomorphic, rational matrix Lax pair for $q - P_{VI}$   |
| 11:10-11:55 | Xin Wang          | Elliptic Garnier systems from 6d $Sp(N)$ gauge theories  |
| 12:00-13:30 | Lunch             |  |
| Chair       | Yik Man Chiang    |  |
| 14:30-15:15 | Tomoyuki Takenawa | Initial value spaces of 4-dimensional Garnier systems  |
| 15:15-16:00 | Teruhisa Tsuda    | Birational Weyl group actions and $q$ -Painlevé equations via mutation combinatorics in cluster algebras             |
| 16:00-16:30 | Coffee Break      |  |
| Chair       | Di Yang           |  |
| 16:30-17:15 | Alexander Stokes  | On the geometry of a 4-dimensional extension of the $A_1^{(1)}q$ -Painlevé I equation                                |
| 17:15-18:00 | Anton Dzhamay     | Discrete Painlevé equations from geometric deautonomization of QRT maps  |
| 18:00-19:30 | Dinner            |  |



**April 8, 2025 - Tuesday**

| Time        | Name                       | Title   |
|-------------|----------------------------|---|
| 7:30~8:30   | Breakfast                  |   |
| Chair       | Da-jun Zhang               |   |
| 9:00-9:50   | Andrei Marshakov           | Higher q-Painlevé equations from reduced cluster integrable systems                     |
| 9:50-10:20  | Group Photo & Coffee Break |   |
| Chair       | Ruguang Zhou               |   |
| 10:20-11:05 | Oleg Chalykh               | Character varieties, DAHAs, and integrable systems                                      |
| 11:10-11:55 | Di Yang                    | Combinatorics and large genus asymptotics of the Brézin-Gross-Witten numbers            |
| 12:00-13:30 | Lunch                      |   |
| Chair       | Xin Wang                   |   |
| 14:30-15:15 | Rod Halburd                | Painlevé and higher Painlevé equations in complex function theory                       |
| 15:15-16:00 | Hajime Nagoya              | On bilinear equations for irregular conformal blocks and quantum Painlevé tau functions |
| 16:00-16:30 | Coffee Break               |   |
| Chair       | Xiangke Chang              |   |
| 16:30-17:15 | Guofu Yu                   | Cauchy bi-orthogonal polynomials and integrable lattices                                |
| 17:15-18:00 | Ying-ying Sun              | A recent study on the CKP equation  |
| 18:00-19:30 | Dinner                     |   |

**April 9, 2025 - Wednesday**

| Time        | Name                        | Title   |
|-------------|-----------------------------|---|
| 7:30~8:30   | Breakfast                   |   |
| Chair       | Ying-ying Sun               |   |
| 9:00-9:50   | Xiangke Chang               | Deformed orthogonal functions and integrable lattices                                 |
| 9:50-10:20  | Coffee Break                |   |
| Chair       | Qian Tang                   |   |
| 10:20-11:05 | Yik Man Chiang              | D-modules approach to discrete special functions                                      |
| 11:10-11:55 | Zhaowen Yan                 | The application of symmetric functions in universal character-type integrable systems |
| 12:00-13:30 | Lunch                       |   |
| Chair       | Free Discussion 13:30-17:00 |   |
| 14:30-15:15 |                             |   |
| 15:15-16:00 |                             |   |
| 16:00-16:30 |                             |   |
| Chair       |                             |   |
| 16:30-17:15 |                             |   |
| 17:15-18:00 |                             |   |
| 18:00-20:00 | Banquet                     |   |

**April 10, 2025 - Thursday**

| Time        | Name                       | Title   |
|-------------|----------------------------|---|
| 7:30~8:30   | Breakfast                  |   |
| Chair       | Cheng Zhang                |   |
| 9:00-9:50   | Qian Tang                  | Isomonodromy equations and the inverse monodromy problem  |
| 9:50-10:20  | Group Photo & Coffee Break |   |
| Chair       | Zhaowen Yan                |   |
| 10:20-11:05 | Yuancheng Xie              | Asymptotics of Okubo system   |
| 11:10-11:55 | Xiaomeng Xu                | Painlevé equations, crystal basis, Riemann-Hilbert problems   |
| 12:00-13:30 | Lunch                      |   |
| Chair       | Zhiwei Wu                  |   |
| 14:30-15:15 | Ruguang Zhou               | Integrable nonlocal finite-dimensional Hamiltonian systems related to the Ablowitz-Kaup-Newell-Segur system |
| 15:15-16:00 | Cheng Zhang                | On compatible ODE systems   |
| 16:00-16:30 | Coffee Break               |   |
| Chair       | Xiaomeng Xu                |   |
| 16:30-17:15 | Lun Zhang                  | On the Painlevé II hierarchy and their applications   |
| 17:15-18:00 |                            |   |
| 18:00-19:30 | Dinner                     |   |

**April 11, 2025 - Friday**

| Time        | Name          | Title   |
|-------------|---------------|---|
| 7:30~8:30   | Breakfast     |   |
| Chair       | Yuancheng Xie |   |
| 9:00-9:50   | Maksim Pavlov | A natural deformation of solutions for semi-Hamiltonian hydrodynamic type systems to a class of dispersive integrable systems |
| 9:50-10:20  | Coffee Break  |   |
| Chair       | Frank Nijhoff |   |
| 10:20-11:05 | Zhiwei Wu     | Darboux transformations for the KdV-type hierarchies  |
| 11:10-11:55 |               |   |
| 12:00-13:30 | Lunch         |   |
| Chair       |               |   |
| 14:30-15:15 |               |   |
| 15:15-16:00 |               |   |
| 16:00-16:30 |               |   |
| Chair       |               |   |
| 16:30-17:15 |               |   |
| 17:15-18:00 |               |   |
| 18:00-20:00 | Banquet       |   |

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## ***Titles and Abstracts***

## On isomonodromic deformation problems on the torus associated with Lax pairs for elliptic integrable lattice systems

**Frank Nijhoff**

The University of Leeds, Shanghai University

I will introduce a class of Lax pairs for elliptic lattice equations comprising Q4 and a version of the lattice Landau-Lifschitz equation, and show how to deautonomise them to obtain isomonodromic deformation problems on the torus. I will derive the associated constitutive relations, and analyze them in the simplest cases to show they exhibit the behaviour of discrete Painleve type equations.

## A holomorphic, rational matrix Lax pair for $q - P_{VI}$

**Robert Conte**

École normale supérieure de Paris-Saclay, The University of Hong Kong

The matrix Lax pairs of Jimbo and Miwa [4] for  $P_{VI}$  and of Jimbo and Sakai [5] for  $q - P_{VI}$  do not exist when respectively  $\theta_\infty^2 = 1$  and  $\kappa_1 = \kappa_2$ , in their notation, i.e. when the residue at infinity, assumed diagonal, has a double eigenvalue.

For  $P_{VI}$ , such a matrix Lax pair holomorphic in the four monodromy exponents is the moving frame of Bonnet surfaces [1, 3], the result being a Jordan matrix instead of a diagonal one for the residue at infinity.

For  $q - P_{VI}$ , by lack of discrete Bonnet surfaces, a Jordan matrix assumption allows us to obtain an explicit matrix Lax pair for any  $\kappa_1, \kappa_2$ , holomorphic in all the discrete monodromy exponents, rational in the two dependent variables of  $q - P_{VI}$ , whose residue at infinity is  $\begin{pmatrix} A_{\infty,11} & 1 \\ 0 & A_{\infty,22} \end{pmatrix}$ .

### References

- [1] R. Conte, C.R. Math. Acad. Sci. Paris **342** (2017) 40-44.
- [2] R. Conte, J. Math. Phys. **58** 103508 (2017) (31 pp).
- [3] R. Conte and M. Musette, The Painlevé Handbook, (Springer Nature, Switzerland, 2020).
- [4] M. Jimbo and T. Miwa, Physica D **2** (1981) 407-448.
- [5] M. Jimbo and H. Sakai, Lett. Math. Phys. **38** (1996) 145-154.

## Elliptic Garnier systems from 6d $Sp(N)$ gauge theories

**Xin Wang**

University of Science and Technology of China

Supersymmetric gauge theories have close connections to integrable systems. For instance, one can explicitly construct supersymmetric defects in gauge theory, which act as eigenfunctions of certain differential or difference operators. In our work, we construct the 1/2-BPS defect for 6d  $Sp(N)$  gauge theories and we find the corresponding operators are related to elliptic Garnier systems.

## Initial value spaces of 4-dimensional Garnier systems

**Tomoyuki Takenawa**

Tokyo University of Marine Science and Technology

We will introduce how to construct initial value spaces in higher dimensions through the construction of initial value spaces of the 4-dimensional Garnier systems. Starting from discrete symmetries, we construct initial value spaces as compact rational varieties. Discrete symmetries act as pseudo-isomorphisms on initial value spaces and as isomorphisms on the Neron-Severi dual lattice.

## Birational Weyl group actions and $q$ -Painlevé equations via mutation combinatorics in cluster algebras

**Teruhisa Tsuda**

Aoyama Gakuin University

A cluster algebra is an algebraic structure generated by operations of a quiver (a directed graph) called the mutations and their associated simple birational mappings. By using a graph-combinatorial approach, we present a systematic way to derive a tropical, i.e. subtraction-free birational, representation of Weyl groups from cluster algebras. Our result provides a broad class of Weyl group actions including previously known examples acting on certain rational varieties and hence it is relevant to  $q$ -Painlevé equations and their higher-order extensions. Key ingredients of the argument are the combinatorial aspects of the reflection associated with a cycle subgraph in the quiver. We also discuss symplectic structures of the discrete dynamical systems thus obtained. This talk is based on a joint work with Tetsu Masuda and Naoto Okubo.

## On the geometry of a 4-dimensional extension of the $A_1^{(1)}$ - $q$ -Painlevé I equation

**Alexander Stokes**

Waseda University

The Sakai scheme defines and classifies discrete Painlevé equations in terms of associated generalised Halphen surfaces. However, there is not yet a similar framework for higher-dimensional analogues of discrete Painlevé equations. While a mechanism for associating rational varieties to such higher discrete Painlevé equations via a kind of regularisation of singularities is known, it is not yet clear what the general definition of higher-dimensional analogues of generalised Halphen surfaces should be - we need more examples to inform the development of a general theory. To this end, in this talk we report on the geometry of a 4-dimensional integrable mapping which extends the autonomous form of a  $q$ -Painlevé I equation with surface type  $A_7^{(1)}$  and symmetry type  $A_1^{(1)}$  in the Sakai classification. We construct a rational variety to which the mapping lifts to a pseudoautomorphism, obtained from  $(\mathbb{P}^1)^4$  by blowing-up along 28 subvarieties. We deform this into a family of varieties which admits an action of an extended affine Weyl group by pseudo-automorphisms of the family and use it to obtain two 4-dimensional analogues of discrete Painlevé equations, one of which is a deautonomisation of the original autonomous mapping. Based on joint work with Adrian Stefan Carstea and Tomoyuki Takenawa

## Discrete Painlevé equations from geometric deautonomization of QRT maps

Anton Dzhamay  
BIMSA

In this talk we consider some examples of discrete Painlevé equations that can be obtained from a given QRT map using the technique of geometric deautonomization. One common interesting feature of such equations is that they often correspond to quasi-translations, or the elements of infinite order in the corresponding affine Weyl group whose certain power is a translation. Such elements often become translations if one considers a smaller affine Weyl subgroup, the phenomena that is known as the projective reduction.

## Higher $q$ -Painlevé equations from reduced cluster integrable systems

Andrei Marshakov  
Skolkovo Institute of Science and Technology

I plan to start with the explanation how  $q$ -difference Painlevé equations and their generalizations arise from deautonomization of the Goncharov-Kenyon integrable systems on Poisson cluster varieties. Equivalent integrable systems are related by mutations in a dual cluster structure, and the  $q$ -Painlevé cases are distinguished to be self-dual in this picture. In order to complete the story and fill the gap with the higher  $q$ -Painlevé cases we need to extend the Goncharov-Kenyon class of integrable systems by their cluster Hamiltonian reductions.

## Character varieties, DAHAs, and integrable systems

Oleg Chalykh  
The University of Leeds

We study certain  $GL(2n)$ -character varieties of the 4-punctured sphere that can be viewed as the monodromy spaces for higher Painlevé-VI. In rank 2 (for  $n = 1$ ), these are a family of affine cubic surfaces well-known from Painlevé theory. We relate these varieties to DAHAs, and use this relation to interpret these varieties as natural phase spaces for the classical trigonometric Van Diejen's system. This is a joint work with Brad Ryan (Leeds), [arxiv.org:2410.23456](https://arxiv.org/abs/2410.23456).

## Combinatorics and large genus asymptotics of the Brézin-Gross-Witten numbers

Di Yang  
University of Science and Technology of China

We discover and prove the uniform large genus leading asymptotics of certain normalized Brézin-Gross-Witten numbers, and give a new proof of the polynomiality phenomenon for the large genus. An application to the Painlevé II hierarchy is also obtained. The talk is based on a joint work with Jindong Guo, Paul Norbury and Don Zagier.



## **Painlevé and higher Painlevé equations in complex function theory**

**Rod Halburd**

University College London

We discuss a natural class of value distribution problems for meromorphic functions and show that their solutions are characterised by Painlevé-type equations, including higher Painlevé equations. These problems arise naturally when determining all meromorphic solutions of differential equations whose general solutions are not meromorphic and proving that all such solutions have been found. However, these value distribution results are also of independent interest.

## **On bilinear equations for irregular conformal blocks and quantum Painlevé tau functions**

**Hajime Nagoya**

Kanazawa University

It is well known that the tau functions of Painlevé equations satisfy bilinear equations. We explain that bilinear equations for Virasoro irregular conformal blocks with the central charge  $c = 1$  naturally coincide with those of Painlevé equations. In general  $c$ , counterparts of irregular conformal blocks are bilinear equations for quantum Painlevé tau functions.

## **Cauchy bi-orthogonal polynomials and integrable lattices**

**Guofu Yu**

Shanghai Jiaotong University

We first consider the generalised two-parameter Cauchy two-matrix model and the corresponding integrable lattice equation. In the second part, we construct discrete spectral transformations for Cauchy orthogonal polynomials, and find its corresponding discrete integrable systems. It turns out that the normalization factor of Cauchy orthogonal polynomials acts as the tau-function of the discrete CKP equation with a discrete Gram determinant structure.

## **A recent study on the CKP equation**

**Ying-ying Sun**

University of Shanghai for Science and Technology

In this talk, I will show the generalized Cauchy matrix scheme for both the continuous and lattice CKP equation which can help us get their soliton solutions and some related exact solutions. Moreover, an elliptic direct linearization scheme is set up for the lattice CKP equation. As a consequence, the lattice CKP equation in the elliptic form together with its Lax pairs can be derived. By selecting appropriate measures and domains for the singular linear integral equation in the scheme, novel elliptic multi-soliton solutions of the lattice CKP equation are presented as well.

## **Deformed orthogonal functions and integrable lattices**

**Xiangke Chang**

Chinese Academy of Sciences

Since the 1990s, the theory of orthogonal polynomials has been increasingly playing an important role in the studies of Toda type lattices, peakon dynamical systems of the Camassa-Holm type, as well as specific Painlevé equations. These integrable lattices can be derived according to deformations of orthogonal functions, directly or indirectly. This talk is devoted to exploring some of related works with focus on our recent results for some new orthogonality. In particular, some higher order discrete Painlevé-type equations are obtained.

## **D-modules approach to discrete special functions**

**Yik Man Chiang**

The Hong Kong University of Science & Technology

It is well-known that D-modules techniques have been adopted to drive hypergeometric type identities. We show that it is also possible to use D-modules to describe classical special functions in general. Different manifestations of the "D" will then enable one to identify and perhaps to discover discrete analogues of classical special functions. Examples will be constructed to illustrate this idea.

## **The application of symmetric functions in universal character-type integrable systems**

**Zhaowen Yan**

Inner Mongolia University

The universal character hierarchy is a generalization of the KP hierarchy. In this talk, we will discuss the tau functions of universal character-type integrable systems using the quantum field presentation of certain symmetric functions. Furthermore, we will present and prove the Pieri rules for the universal character and the symplectic universal character.

## **Isomonodromy equations and the inverse monodromy problem**

**Qian Tang**

The University of Hong Kong

In this talk, we will introduce our recent work on some isomonodromy equations. We can regard it as a higher Painlevé equations. Our main result is the asymptotic behavior of its generic solution; we also provide its series solution starting from the monodromy data and use this method to study the inverse monodromy problem.

## Asymptotics of Okubo system

**Yuancheng Xie**  
Beijing University

Okubo system is the Birkhoff canonical form for differential equations of Poincaré rank 1. It appears in the construction of Dubrovin's Frobenius manifold and also Bridgeland stability condition etc. Its isomonodromy deformation is an important source of higher rank Painlevé systems. In this talk I will outline a way to study the asymptotic behavior of its solutions at some boundary points using the Riemann-Hilbert techniques.

## Painlevé equations, crystal basis, Riemann-Hilbert problems

**Xiaomeng Xu**  
Beijing University

The Painlevé VI equation is equivalent to the isomonodromy deformation equation of meromorphic linear systems of rank 3 with a second order pole. Its boundary condition, monodromy formula and many global properties were given in 1980s. The isomonodromy equations of meromorphic linear systems of rank  $n$  with a second order pole can be seen as higher Painlevé equations. This talk recalls the long time asymptotics and monodromy formula for such higher Painlevé equations, and then explains how the analysis formula is related to various structures in the representation theory of quantum groups, like crystal basis, Gelfand-Tsetlin basis, and combinatorics of Young tableaux.

## Integrable nonlocal finite-dimensional Hamiltonian systems related to the Ablowitz-Kaup-Newell-Segur system

**Ruguang Zhou**  
Jiangsu Normal University

The method of nonlinearizing the Lax pair represents an efficient approach for decomposing  $(1 + 1)$ -dimensional soliton equations into finite-dimensional integrable Hamiltonian systems. This method is useful for constructing the algebra-geometric solutions of soliton equations. In this talk, we will extend the method of nonlinearizing the Lax pair to the Ablowitz-Kaup-Newell-Segur (AKNS) equation under space-inverse reductions. We obtain a new class of finite-dimensional Hamiltonian systems. These systems are nonlocal, as they involve the inverse of the space variable. For these nonlocal Hamiltonian systems, we demonstrate that they maintain Liouville integrability and can be linearized on the Jacobi variety. Additionally, we show how to construct the algebra-geometric solutions of the AKNS equation with space-inverse reductions by means of our nonlocal finite-dimensional Hamiltonian systems. As an application, we obtain algebra-geometric solutions of the AKNS equation with Dirichlet and Neumann boundary conditions, as well as algebra-geometric solutions of the nonlocal nonlinear Schrödinger (NLS) equation.

## On compatible ODE systems

**Cheng Zhang**  
Shanghai University

Dubrovin constructed an infinite compatible ODE system that is equivalent to the KdV hierarchy. The construction is based on a r-matrix structure. In this talk, I will present some generalization of Dubrovins approach. The talk is based on a joint work with Di Yang, Zejun Zhou.

## On the Painlevé II hierarchy and their applications

**Lun Zhang**  
Fudan University

In this talk, I will report my recent works on the asymptotic aspects of the Painlevé II hierarchy. Their applications in random matrix theory and integrable differential equations will also be discussed.

## A natural deformation of solutions for semi-Hamiltonian hydrodynamic type systems to a class of dispersive integrable systems

**Maksim Pavlov**  
Shandong University of Science and Technology

In this talk we consider dispersive integrable systems and their dispersionless limits. In a dispersionless limit these systems are integrable by the Tsarev Generalized Hodograph Method, which determines a general solution. It was known that reflectionless potentials (multi-soliton solutions, rational solitons, multi-phase solutions, etc.) have no a dispersionless limit. In this talk we give the answer to the natural question for dispersive integrable systems: is it possible to find exact analytic solutions, which possess a dispersionless limit? Yes, this class also is well known in the theory of dispersive integrable systems: "isomonodromic deformations". We illustrate our construction by a set of such particular solutions determined by corresponding Lax representations. in 2D we consider the Kaup-Boussinesq system, while in 3D we investigate the Mikhalev system.

## Darboux transformations for the KdV-type hierarchies

**Zhiwei Wu**  
Sun Yat-sen University

The KdV-type hierarchies can be related to affine Kac-Moody algebras, and some of them have natural geometric correspondence. In this talk, we will discuss the Darboux transformations for KdV-type hierarchies in terms of loop group factorization. Permutability formulas are derived to construct explicit solutions. We will give some explicit examples, such as the Boussinesq and Kaup "CKupersmidt equations. They are associated to different types of affine Kac-Moody algebra and related to curve flows under certain group actions.

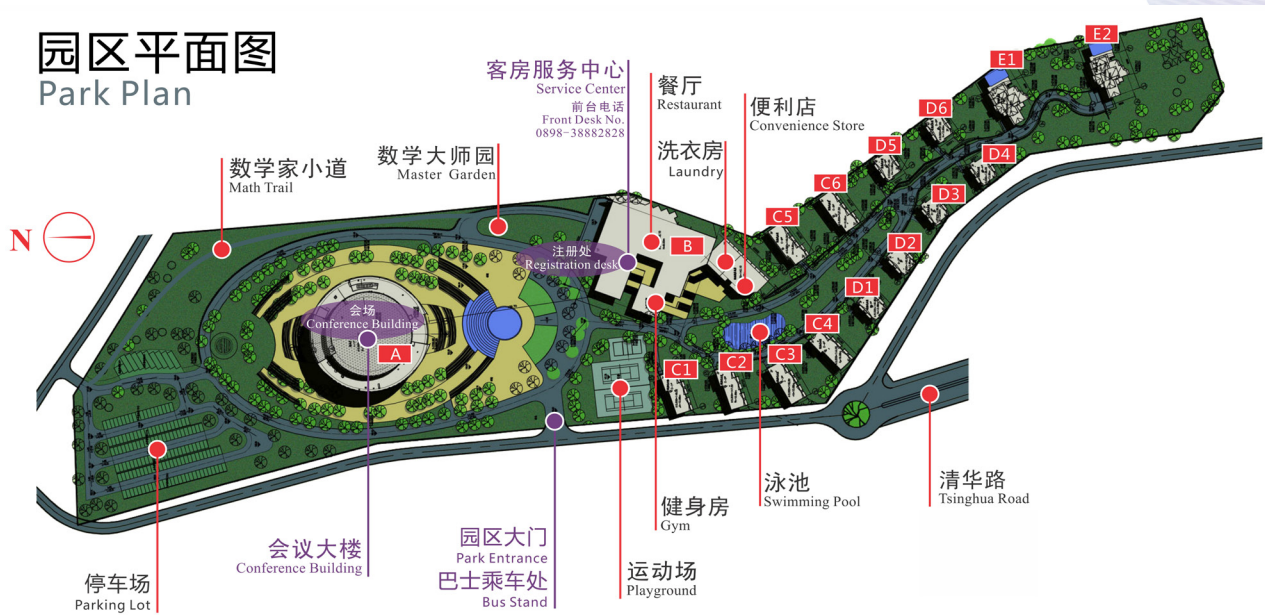
***Welcome to TSIMF***



The facilities of TSIMF are built on a 23-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 29,000 square meter that includes state-of-the-art conference facilities (over 10,000 square meter) to hold many international workshops simultaneously, two reading rooms of library, a guest house (over 10,000 square meter) and the associated catering facilities, a large swimming pool, gym and sports court and other recreational facilities.

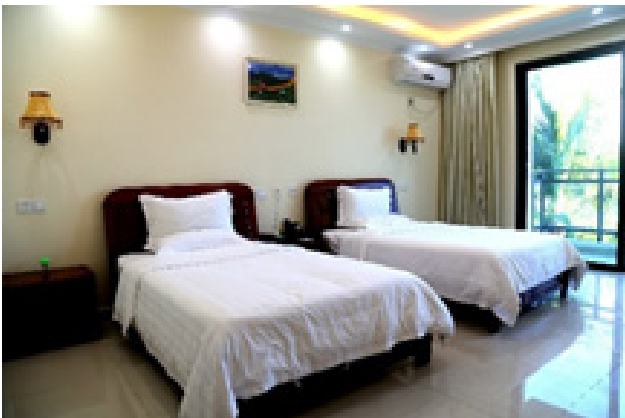
Management Center of Tsinghua Sanya International Forum is responsible for the construction, operation, management and service of TSIMF. The mission of TSIMF is to become a base for scientific innovations, and for nurturing of innovative human resource; through the interaction between leading mathematicians and core research groups in pure mathematics, applied mathematics, statistics, theoretical physics, applied physics, theoretical biology and other relating disciplines, TSIMF will provide a platform for exploring new directions, developing new methods, nurturing mathematical talents, and working to raise the level of mathematical research in China.

## About Facilities



## Registration

Conference booklets, room keys and name badges for all participants will be distributed at the front desk. Please take good care of your name badge. It is also your meal card and entrance ticket for all events.



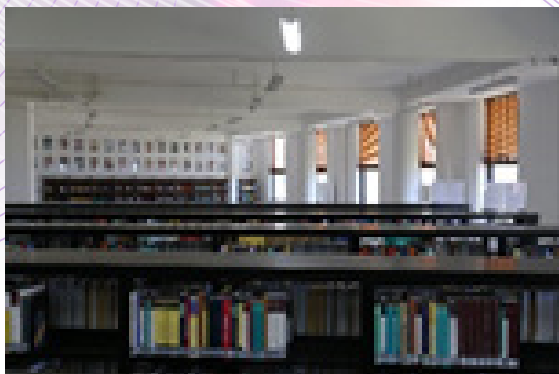
## Guest Room

All the rooms are equipped with: free Wi-Fi (Password:tsimf123), TV, air conditioning and other utilities.

Family rooms are also equipped with kitchen and refrigerator.



## Library



**Opening Hours: 09:00am-22:00pm**

TSIMF library is available during the conference and can be accessed by using your room card. There is no need to sign out books but we ask that you kindly return any borrowed books to the book cart in library before your departure.



In order to give readers a better understanding of the contributions made by the Fields Medalists, the library of Tsinghua Sanya International Mathematics Forum (TSIMF) instituted the Special Collection of Fields Medalists as permanent collection of the library to serve the mathematical researchers and readers.

So far, there are 271 books from 49 authors in the Special Collection of Fields Medalists of TSIMF library. They are on display in room A220. The participants are welcome to visit.



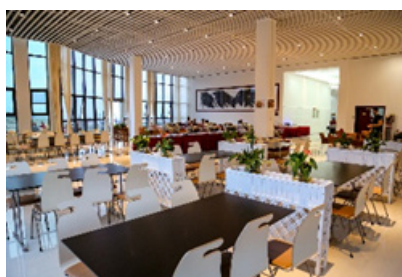
## Restaurant

All the meals are provided in the restaurant (Building B1) according to the time schedule.

**Breakfast 07:30-08:30**

**Lunch 12:00-13:30**

**Dinner 17:30-19:00**





## Laundry

### Opening Hours: 24 hours

The self-service laundry room is located in the Building(B1).



## Gym

### Opening Hours: 24 hours

The gym is located in the Building 1 (B1), opposite to the reception hall. The gym provides various fitness equipment, as well as pool tables, tennis tables etc.



## Playground

Playground is located on the east of the central gate. There you can play basketball, tennis and badminton. Meanwhile, you can borrow table tennis, basketball, tennis balls and badminton at the reception desk.

## Swimming Pool

Please enter the pool during the open hours, swimming attire and swim caps are required, if you feel unwell while swimming, please stop swimming immediately and get out of the pool. The depth of the pool is 1.2M-1.8M.

**Opening Hours: 13:00-14:00 18:00-21:00**



## Free Shuttle Bus Service at TSIMF

We provide free shuttle bus for participants and you are always welcome to take our shuttle bus, all you need to do is wave your hands to stop the bus.

Destinations: Conference Building, Reception Room, Restaurant, Swimming Pool, Hotel etc.



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## Contact Information of Administration Staff

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